Baghouse Failures: Applying the Weibull Distribution to Estimate Bag Failure Emissions as a Function of Time

Paper 2013-12-112-AWMA

R. Ryan Dupont
Utah Water Research Laboratory, Utah State University, Department of Civil and Environmental Engineering, 8200 Old Main Hill, Logan, UT 84322-8200
J. D. McKenna
ETS, Inc., 1401 Municipal Road, NW, Roanoke, VA 24012-1309
L. Theodore
Theodore Tutorials, 5 Fairview Ave, East Williston NY 11596

ABSTRACT

An earlier paper detailed calculations associated with the effect of bag failure on baghouse outlet loading. This paper extends this development to include the estimation of these baghouse emissions as a function of time using the Weibull distribution to predict the time course of bag failures.

The highest source of maintenance and cost for baghouses is generally the filter bags. All bag sets have a finite lifetime that will vary by application, installation, operating parameters, fabric type, etc. Typically, a few bags will fail initially or after a short period of operation as a result of installation damage or manufacturing defects. The failure rate should then remain low until the operating life of the bags is reached unless a unique failure mode is present within the system. The failure rate then increases, normally at a near exponential rate. Industry often describes this type of failure rate behavior via a Weibull distribution similar to a *bathtub* curve.

A short review of the earlier paper is followed by a discussion of the Weibull distribution and its application to baghouse operation and maintenance problems. The procedures presented allow one to both calculate the effect of bag failure on particulate discharge from baghouses, as well as estimate the number of and times when bags will fail. In addition to reducing operation and maintenance for baghouse operators, the method allows for quick response to malfunctions associated with bag failures.

Four illustrative examples complement the presentation.

INTRODUCTION

An earlier paper[1] detailed calculations associated with the effect of bag failures on baghouse outlet loading. This paper extends this development to include the estimation of these baghouse emissions as a function of time using the Weibull distribution to predict the time course of bag failures. A short review of the earlier paper is followed by a discussion of the Weibull distribution and its application to baghouse operation and maintenance problems.

The highest source of maintenance and cost for baghouses is generally the filter bags. All bag sets have a finite lifetime that will vary by application, installation, operating parameters, fabric type, etc. Typical causes of bag failure are:

1. High localized gas velocity (due to gas maldistribution)
2. Metal-to-cloth abrasion
3. Chemical attack
4. Bag-to-bag abrasion
5. Inlet velocity abrasion (on inside-out cleaning configuration)
6. Accidents
7. Upset conditions (e.g., temperature excursions)
8. Thread mismatch
9. Cuff mismatch
10. Improper installation

In addition, each bag in a set may have a different life as a result of fabric quality, bag manufacturing tolerances, location in the collector, and variation in the bag-cleaning mechanism. Any one or a combination of these factors can cause bags to fail. This means that a baghouse will experience a series of intermittent bag failures until the failure rate requires total bag replacement. Typically, a few bags will fail initially or after a short period of operation as a result of installation damage or manufacturing defects. The failure rate should then remain very low until the operating life of the bags is reached unless a unique failure mode is present within the system. The failure rate then increases, normally at a near exponential rate. Industry often describes this type of failure rate behavior via a Weibull distribution similar to a bathtub curve[2,3], and details of this failure distribution are presented below.

The importance of when to replace a broken bag will depend on the nature of the emission, the type of collector and the resultant effect on outlet emissions. In "inside bag collection" type collectors with shaker and reverse-air cleaning, it is very important that dust leaks be stopped as quickly as possible to prevent adjacent bags from being abraded by jets of dust being emitted from the broken bag, causing a "domino" effect of bag failure. "Outside bag collection" systems with pulse jet cleaning do not have this problem, and the speed of repair for these systems is driven by whether the outlet opacity has exceeded its regulated limit. Often, it will take several broken bags to create an opacity problem, and a convenient maintenance schedule can be employed in these "outside bag collection' systems instead of requiring emergency maintenance.

In either type of collector, the location of the broken bag(s) has to be determined and corrective action taken. In a non-compartmentalized unit, this requires system shutdown and visual inspection. In inside bag collectors, bags often fail close to the bottom of the bag, near the tube sheet. Accumulation of dust on the tube sheets, the holes themselves, or unusual dust patterns on the outside of the bags often occurs. Other probable bag failure locations in inside bag collectors with reverse-air cleaning are near anti-collapse rings or below the top cuff. In inside bag collectors with shaker cleaning, one should inspect the area below the top attachment. Improper bag tensioning can also lead to premature bag failures.

In outside bag collectors, which are normally top-access systems, inspection of the bags themselves is difficult; however, location of the broken bag(s) can normally be accomplished by looking for dust accumulation on top of the tube sheet, on the underside of the top-access door, or on a blowpipe.

EFFECT OF BAG FAILURE ON OUTLET LOADING

The effect of bag failure on baghouse efficiency can be described by the following equations:

$$P_t^* = P_t + P_{tc} \qquad \textbf{(Eq. 1)}$$

$$P_{tc} = \frac{0.582(\Delta P)^{1/2}}{\phi} \qquad \textbf{(Eq. 2)}$$

$$\phi = \frac{q}{LD^2(T+460)^{1/2}} \qquad \textbf{(Eq. 3)}$$

where:

P_t^* = penetration after bag failure
P_t = penetration before bag failure
P_{tc} = penetration correction term; contribution of broken bag to P_t^*
ΔP = pressure drop, in H_2O
ϕ = dimensional parameter
q = volumetric flow rate of contaminated gas, acfm
L = number of broken bags
D = bag diameter, inches
T = temperature, F

Refer to the literature for a detailed development of Equations 1 through 3.[1,3,4]

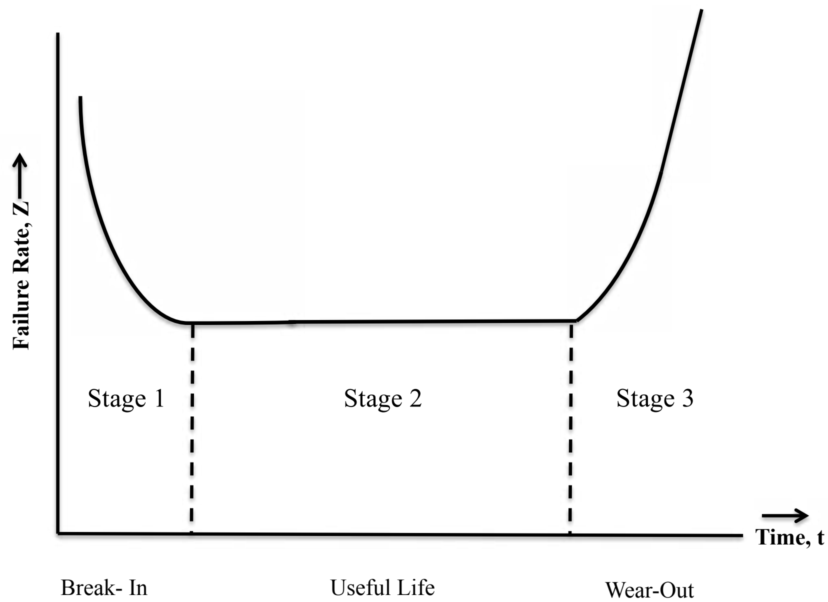
THE WEIBULL DISTRIBUTION[2]

The failure rate of equipment frequently exhibits three stages:

1) a break-in stage with a declining failure rate over time,
2) a useful life stage characterized by a fairly constant failure rate over time, and
3) a wear-out stage characterized by an increasing failure rate with increasing operating time.

Many industrial parts and components, as well as human mortality rates with respect to age, follow this three-stage relationship. A failure rate curve exhibiting these three phases (see Figure 1) has been referred to as the aforementioned bathtub curve.

Figure 1. The Bathtub Failure Rate Curve.



Weibull introduced the distribution, which bears his name principally on empirical grounds, to represent certain life-test data. The Weibull distribution provides a mathematical model of all three stages of the bathtub curve. The model describing the failure rate, Z, which reflects all three of the bathtub stages is:

$$Z(t) = \alpha\beta t^{\beta-1};\ t > 0 \qquad \textbf{(Eq. 4)}$$

where:

α and β = constants
t = time

For $\beta < 1$ the failure rate *Z(t)* decreases with time (Figure 1, Stage 1). For $\beta = 1$ the failure rate is constant and equal to α (Figure 1, Stage 2). For $\beta > 1$ the failure rate increases with time (Figure 1, Stage 3). One can translate the above failure rate equation into a corresponding probability density function (pdf) of *t,* time-to-failure, as shown in Equation 5:

$$f(t) = \alpha\beta t^{\beta-1}\exp\left(\int_0^t \alpha\beta t^{\beta-1}dt\right) = \alpha\beta t^{\beta-1}\exp\left(-\alpha t^{\beta}\right);\ t{>}0, \alpha{>}0, \beta{>}0 \qquad \textbf{(Eq. 5)}$$

Equation 5 defines the pdf of the Weibull distribution.[2,3] The exponential distribution is a special case of the Weibull distribution with $\beta = 1$. The variety of assumptions about failure rate and the probability distribution of time-to-failure that can be accommodated by the Weibull distribution make it especially attractive in describing failure time distributions, perhaps accounting for its widespread use in industrial and process hazard risk applications.[2,3]

Estimates of the values of parameters ∝ and β in Equation 5 can be obtained by using a graphical procedure developed by Bury.[5] This procedure is based on the fact that

$$\ln\left[\ln \frac{1}{1\text{-}F(t)}\right] = \ln(\alpha)+\beta \ln(t) \qquad \textbf{(Eq. 6)}$$

where:

$F(t) = 1 - \exp(-\alpha t^{\beta})$ for $t>0$, and $F(t) = 0$ for $t < 0$ defines the cumulative distribution function of the Weibull distribution

Equation 6 is a linear relationship that can be represented in the form:

$$Y = b + mX \qquad \textbf{(Eq. 7)}$$

where:

$Y = \ln[\ln 1/(1 - F(t))]$
$X = \ln(t)$
$b = \text{intercept} = \ln(\alpha)$
$m = \text{slope} = \beta$

The graphical procedure for estimating ∝ and β on the basis of a sample of n observed values of time-to-failure, t, first involves the ordering of the observations of time-to-failure from smallest (i = 1) to largest (i = *n)* when the value of the i^{th} observation varies from sample to sample. It can be shown that the average value of F(t) for t equal to the value of the i^{th} observation of time-to-failure can be approximated by $i/(n+1)$. One may then plot $\ln[\ln 1/(1 - i/(n+1))]$ against the natural logarithm of the i^{th} observation of time-to-failure, t_i, from $i = 1$ to n. Under the assumption that the time-to-failure has a Weibull distribution, the plotted points lie on a straight line whose slope is β and whose intercept is ln(∝). This procedure for estimating ∝ and β is illustrated in Example 2 below.

ILLUSTRATIVE EXAMPLES OF BAGHOUSE FAILURE CALCULATIONS AND THE APPLICATION OF THE WEIBULL DISTRIBUTION

The following examples[6-9] demonstrate the quantitative application of the Weibull distribution for evaluation and prediction of baghouse bag failures.

Illustrative Example 1 – Estimation of Tolerable Bag Failure Rate for Meeting Baghouse Performance Requirements

DMT Industries owns and operates a 12-yr old 4,000 bag baghouse system consisting of several compartments. The bags are 4 in diameter, and the pressure drop across the system is 7.0 in H_2O. The operating temperature and pressure are 110°F and 1 atm, respectively. The inlet loading to the baghouse is 4.0 gr/ft^3, and the system is 99.91% efficient, assuming that all bags are completely functional. The treated gas flow rate is 770,000 acfm, and the filtering velocity is 420 ft/h. What is the maximum number of bag failures that can be tolerated to ensure a minimum collection efficiency of 98.57% for the baghouse to remain in compliance?

Solution - For this system,

$$P_t = 1 - E = 1\text{-}0.9991\text{=}0.0009 \text{ (before failure)}$$

and

$$P_t^* = 1 - 0.9857 = 0.0143 \text{ (after failure)}$$

Therefore, from Equation 1,

$$P_{tc} = P_t^* - P_t = 0.0143 - 0.0009 = 0.0134$$

The dimensional parameter, ϕ from Equation 2 can be determined from this calculated value of the penetration correction term as follows:

$$\phi = \frac{0.582(\Delta P)^{1/2}}{P_{tc}} = \frac{0.582\ (7.0)^{1/2}}{0.0134} = \frac{1.54}{0.0134} = 114.9 \qquad \textbf{(Eq. 8)}$$

The maximum number of broken bags that can be allowed is determined from this value of ϕ by rearrangement of Equation 3 and solving for L as:

$$L = \frac{q}{\phi D^2 (T+460)^{1/2}} = \frac{770{,}000}{(114.9)*(4^2)*(110+460)^{1/2}} = \frac{770{,}000}{43{,}896} = 17.5 = 18 \text{ bags} \qquad \textbf{(Eq. 9)}$$

Thus, a maximum of 18 bag failures can be tolerated to ensure a minimum baghouse collection efficiency of 98.57%. The above calculation does not provide information on when, i.e., what time, the baghouse will be out of compliance. This issue is addressed in the next three Illustrative Examples.

Illustrative Example 2 – Estimation of Weibull Parameters for Baghouse Bag Failure Rate

Following startup, the time in months to failure of an individual bag in the baghouse described in the previous example was recorded as follows: 0.1, 0.5, 1.2, 3, 17, 20, 23.6, 28, 30.8, 34, 37.5, 39, 40.5, 41.5, and 42.5. If a Weibull distribution applies to these 15 bag failure data, estimate the values of α and β for this Weibull distribution.

Solution – Table 1 is generated from the data presented in the Problem Statement. The first two columns of Table 1 are tabulated from these data. The values in Column 3 are calculated as the natural logarithm of corresponding time-to-failure, t, data in Column 1. Results in Column 4 are generated from the equation ln[ln(1/(1-i/(n + 1))] using corresponding values of order-of-failure, i, in Column 2 and n = 4000 based on the given failure data set. The data in Table 1 indicate a break-in period of approximately 3 months, followed by a period of constant failure rate to approximately 38 months before observing an increasing rate of failure beginning at 39 months. This pattern is typical of baghouse operation,[7] so that the data may be used to generate Weibull distribution coefficients over the three periods of bag failure.

Table 1. Data and Calculations of Weibull Coefficients for Illustrative Example 2.

Time-to-Failure (t), months	Order-of-Failure (i)	ln(t)	$\ln\left[\ln\frac{1}{1-\frac{i}{n+1}}\right]$
0.1	1	-2.3	-8.29
0.5	2	-0.7	-7.60
1.2	3	0.2	-7.20
3	4	1.1	-6.91
17	5	2.83	-6.68
20	6	3.00	-6.50
23.6	7	3.16	-6.35
28	8	3.33	-6.21
30.8	9	3.43	-6.10
34	10	3.53	-5.99
37.5	11	3.62	-5.90
39	12	3.66	-5.81
40.5	13	3.70	-5.73
41.5	14	3.73	-5.65
42.5	15	3.75	-5.58

From the results presented in Table 1, and employing Equation 7, a linear regression was applied with the X variable equal to the values of ln(t) in Column 3, and the Y variable equal to the values of ln[ln(1/(1-i/(n + 1))] in Column 4. The regression analysis was carried out using the regression function in Excel, for the three failure mode periods (0 to 3 months, 17 to 37.5 months, and 39 to 42.5 months).

Figure 2 shows results of the regression analysis for the break-in period (0 to 3 months) for this data set.

From Figure 2, the Weibull coefficients for the Break-In period are as follows: α = exp(-7.322) = 0.000661, β = 0.414. The r^2 value is statistically significant and the linear regression fits the transformed data, suggesting that the Weibull model provides an accurate representation of this portion of the failure data. Table 2 summarizes the results of the Weibull coefficient estimation for the other two portions of the Weibull distribution.

Figure 2. Excel Generated Regression Results for Break-In Period Data from Illustrative Example 2.

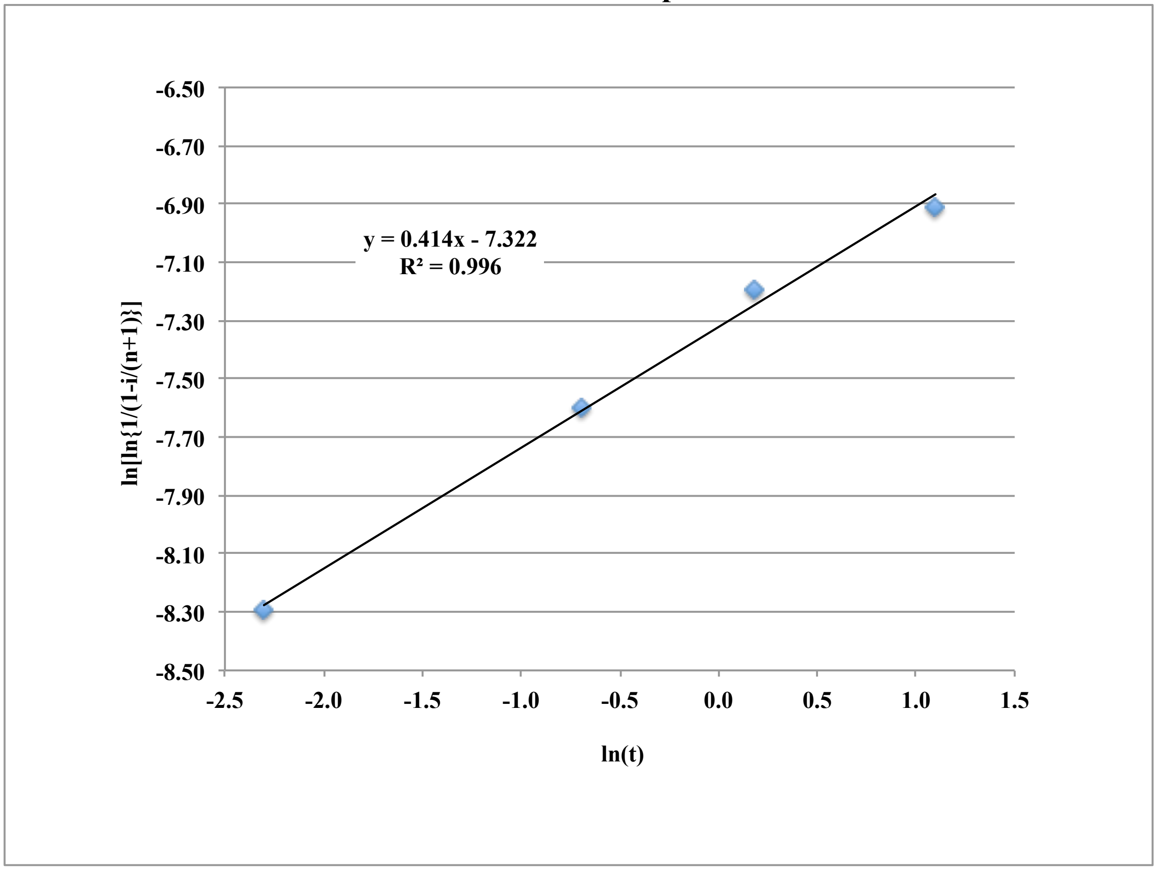


Table 2. Summary Results of Weibull Coefficient Determination from Bag Failure Data Presented in Illustrative Example 2.

Operating Period	α	β	r^2
Break-In (0 to 3 months)	6.61×10^{-4}	0.414	0.996
Useful Life (17 to 37.5 months)	7.83×10^{-5}	0.981	0.998
Wear Out (> 39 months)	2.09×10^{-7}	2.61	0.992

Illustrative Example 3

Assuming the Weibull equation (and the accompanying coefficients generated in the previous Illustrative Example) apply to the baghouse in Illustrative Example 1, *estimate* when the baghouse will be out of compliance.

Solution – To solve this problem, the failure rate, Z(t) from Equation 4, must be calculated. As indicated in Illustrative Example 1, for the baghouse to remain in compliance, no more than 18 bags can be allowed to fail. Based on results from Illustrative Example 2, an 18-bag failure scenario would occur during the Wear-Out period, i.e., > 39 months for this baghouse system. Applying the Weibull distribution coefficients for the Wear-Out period to the calculation of Z(t) results in the following:

$$Z(t) = 2.09\,x\,10^{-7}(2.61)t^{2.61-1};\ t \geq 39\ months \qquad \textbf{(Eq. 10)}$$

Substituting time in months for variable t in Equation 10 allows the estimation of the point failure rate in failures/month that would occur for the bags remaining within the baghouse over time. Multiplying this point failure rate by the number of remaining bags gives the number of bag failures expected at a given time, t. When these expected bag failures are added to the total number of bags that have failed up to the previous time step, the total number of expected failed bags can be estimated. Detailed calculations for the extrapolated time period t = 44 months (1.5 month after the last data point) are presented below.

For t = 42.5 months, this value is substituted in Equation 10 to yield the following:

$$Z(t) = 2.09\,x\,10^{-7}(2.61)\,(42.5)^{2.61-1} = 5.45\,x\,10^{-7}(42.5)^{1.61} = 0.000229/mo$$

For t = 44 months, this value is substituted in Equation 10 to yield the following:

$$Z(t) = 2.09\,x\,10^{-7}(2.61)\,(44)^{2.61-1} = 5.45\,x\,10^{-7}(44)^{1.61} = 0.000243/mo$$

This average failure rate between 42.5 and 44 months is 0.00236/mo, and is applied to the number of bags remaining at t = 42.5 months, which is 3985 bags to yield the following estimated number of failed bags between 42.5 and 44 months:

Predicted number of Bag Failures/mo @ Between Months 42.5 and Month 44
= 0.000236/mo (3985 bags)
= 0.94 Bag Failures/mo Between Month 42.5 and Month 44

The total number of bags estimated to have failed by Month 44 is then determined by:

Probable Total # Bag Failures @ Month 44
= # Bag Failures @ Month 42.5 + (1.5 mo) x (0.94 Bag Failures/mo Between Month 42.5 to Month 44) = 15 + 1.41 = 16.4 Bag Failures @ Month 44

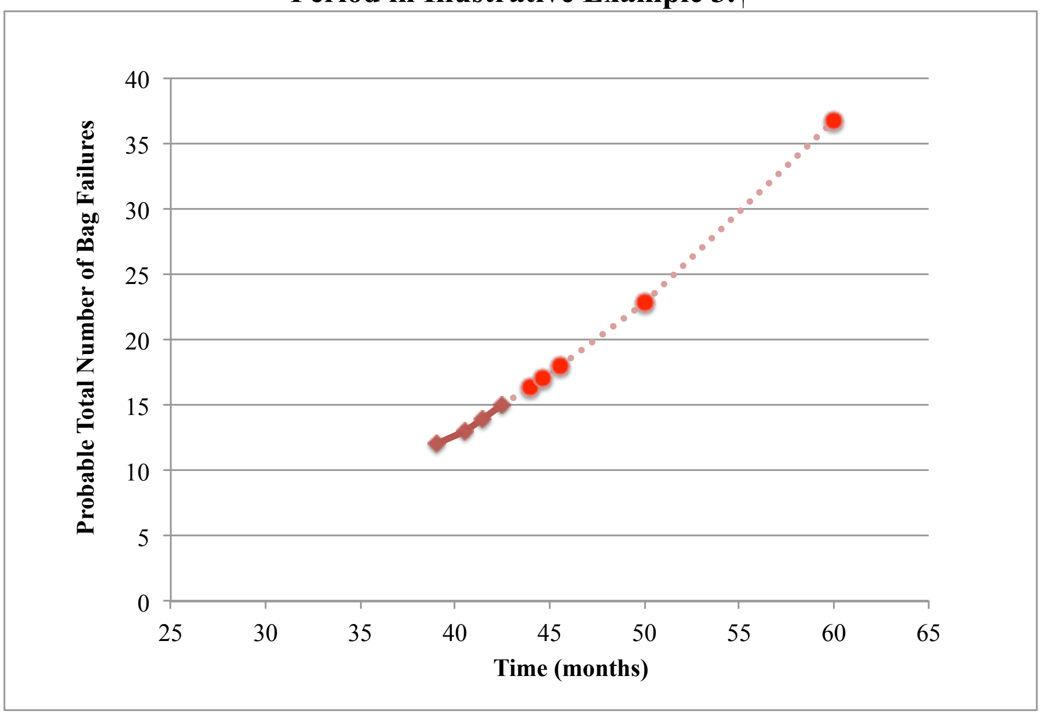
Table 3 summarizes the balance of these incremental monthly calculations for the Wear-Out period for this baghouse system including the point of non-compliance, i.e., a total of 18 bag failures at approximately 45.6 months, out to a 60-month operating period. Figure 3 shows this increasing bag failure rate projected during the Wear-Out period.

Table 3. Summary Calculations for Failure Rate (Z(t)) and Cumulative Bag Failures Over Time Estimated from the Weibull Distribution during the Wear-Out Period.†

Time to Failure, t (mo)	Order of Failure (i)	Z(t), Failures/mo	Predicted # Bag Failures/ΔTime Period	Probable Total # Bag Failures
39	12	0.000200		12
40.5	13	0.000212		13
41.5	14	0.000221	1.20	14
42.5	15	0.000229	1.20	15
44		**0.000243**	**1.41**	**16.4**
44.6		**0.000248**	**0.59**	**17.0**
45.6		**0.000257**	**1.01**	**18.0**
50		**0.000298**	**4.86**	**22.9**
60		**0.000400**	**13.88**	**36.7**

† Table values in bold are extrapolated from the observed bag failure data set presented in Table 1.

Figure 3. Measured and Projected Bag Failures Over Time During the Wear-Out Period in Illustrative Example 3.†



† Projected bag failures indicated by dotted line.

Illustrative Example 4

Estimate the particulate discharge over time from the baghouse described in Illustrative Example 1 as a function of the probable total number of bag failures estimated in Illustrative Example 3.

Solution – Equations 1 through 3 are used to determine the penetration correction term, P_{tc}, that is applied to the initial bag house particulate removal efficiency as individual bags fail over time. Equation 3 is used to dimensional parameter, ϕ, as a function of the number of broken bags, L. This value of ϕ is then substituted into Equation 2 to determine the additional particulate penetration resulting from bag failure, and then the overall particulate penetration is determined from Equation 1. These calculations are demonstrated below for the 17th bag failure predicted to occur at approximately 44.6 months, while Table 4 and Figure 4 summarize the declining particulate removal efficiency predicted over an entire 60-month operating period for this baghouse system.

For 17 failed bags, L = 17. Substituting this value into Equation 3 for the baghouse operating conditions presented in Illustrative Example 1 yields the following:

$$\phi = \frac{\mathbf{q}}{\mathbf{LD^2(T+460)^{1/2}}} = \frac{\mathbf{770{,}000}}{\mathbf{(17)(4)^2(110+460)^{1/2}}} = \frac{\mathbf{770{,}000}}{\mathbf{6510}} = 118$$

This dimensionless parameter is then substituted into Equation 2 to generate a penetration correction factor as follows:

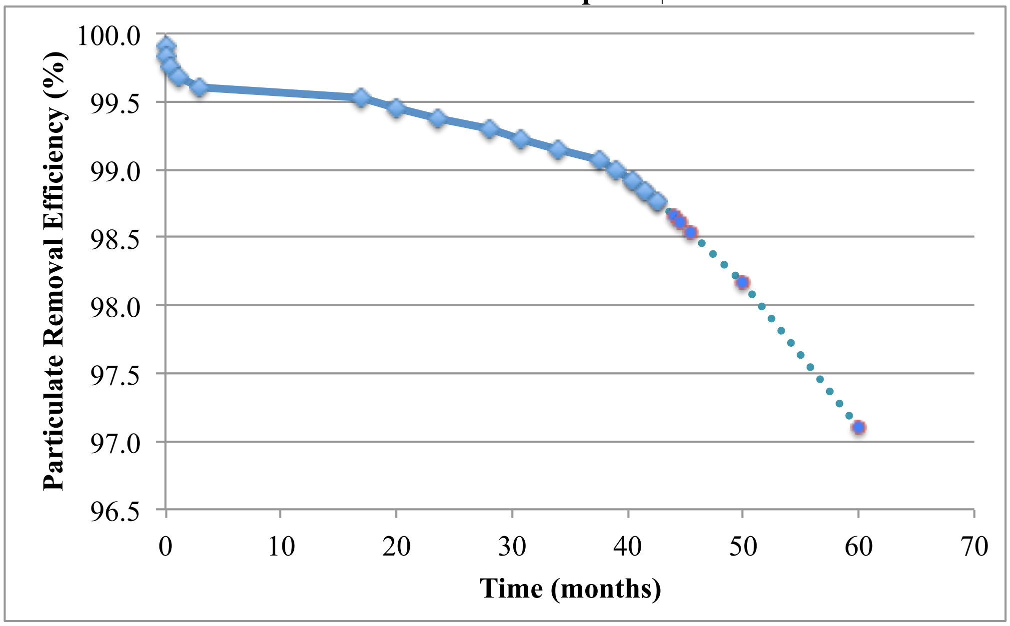
$$P_{tc} = \frac{0.582(\Delta P)^{1/2}}{\phi} = \frac{0.582\,(7)^{1/2}}{118} = 0.0130$$

Table 4. Summary Calculations for Particulate Removal for Baghouse System from Illustrative Example 4.†

Number of Failed Bags, L	Time to Failure (months)	Dimensionless Parameter,	Penetration Correction Term, P_{tc}	Penetration After Bag Failure	Particulate Removal Efficiency, E
0	0	0		0.0009	99.91
1	0.1	2016	0.0008	0.0017	99.83
2	0.5	1008	0.0015	0.0024	99.76
3	1.2	672	0.0023	0.0032	99.68
4	3	504	0.0031	0.0040	99.60
5	17	403	0.0038	0.0047	99.53
6	20	336	0.0046	0.0055	99.45
7	23.6	288	0.0053	0.0062	99.38
8	28	252	0.0061	0.0070	99.30
9	30.8	224	0.0069	0.0078	99.22
10	34	202	0.0076	0.0085	99.15
11	37.5	183	0.0084	0.0093	99.07
12	39	168	0.0092	0.0101	98.99
13	40.5	155	0.0099	0.0108	98.92
14	41.5	144	0.0107	0.0116	98.84
15	42.5	134	0.0115	0.0124	98.76
16.5	**44**	**123**	**0.0125**	**0.0134**	**98.66**
17.0	**44.6**	**119**	**0.0130**	**0.0139**	**98.61**
18.0	**45.6**	**112**	**0.0138**	**0.0147**	**98.53**
22.9	**50**	**88**	**0.0175**	**0.0184**	**98.16**
36.7	**60**	**55**	**0.0281**	**0.0290**	**97.10**

† Table values in bold are based on extrapolated bag failure data.

Figure 4. Particulate Removal Efficiency Over Time for Baghouse System from Illustrative Example 4.†



† Projected particulate removal efficiency results indicated by dotted line.

The penetration after 17 bag failures is then calculated as follows using Equation 1:

$$P_t^* = P_t + P_{tc} = 0.0009 + 0.0130 = 0.0139$$

Finally, the resulting particulate removal efficiency is determined as:

$$\text{Particulate removal efficiency} = 1 - P_t^* = 1 - 0.0139 = 0.9861 = 98.61\%$$

SUMMARY

Procedures presented in an earlier paper by McKenna, Clark and Theodore[1] for detailed calculations associated with the effect of bag failures on baghouse outlet loading were extended in this paper to include the estimation of baghouse emissions as a function of time through the use of the Weibull distribution to predict the time course of bag failures. Illustrative examples were used to demonstrate these calculations and to develop Weibull distribution coefficients describing bag failure rates during Break-In, Useful Life, and Wear-Out periods of baghouse operations. The Weibull distribution during the Wear-Out period was used to predict the time to failure of bags past the present observation time period and to estimate the time when particulate emission non-compliance will occur.

The methodology presented provides a straightforward means of predicting future bag failures, allowing for a timely response to avoid compliance issues and reduce operational and maintenance problems.

REFERENCES

1 McKenna, J.; Clark, C.; Theodore, L. *Future Boiler MATS Regulations & Baghouse Emission Concerns*; Paper #64, 105th Annual Air and Waste Management Association Conference, June 19-22; San Antonio, TX, 2012.

2 Shaefer, S.J.; Theodore, L. *Probability and Statistics Applications for Environmental Science;* CRC Press/Taylor & Francis Group: Boca Raton, FL, 2007.

3 Theodore, L; Reynolds, J. "Effect of Bag Failure on Baghouse Outlet Loading;" *J. APCA* **1979**, 29, 870-872.

4 Theodore, L. "Engineering Calculations: Baghouse Specifications and Operations Simplified;" *Chem. Eng. Progress* **2000**, 96, 6, 22.

5 Bury, K.; *Statistical Methods in Applied Sciences*; John Wiley & Sons: Hoboken, NJ, 1975.

6 Theodore, L. (adopted from) *Air Pollution Control Equipment Calculations*; John Wiley & Sons: Hoboken, NJ, 2008.

7 Theodore, L.; Dupont, R. (adopted from) *Environmental Health and Hazard Risk Assessment: Principles and Calculations*; CRC Press/Taylor & Francis Group: Boca Raton, FL, 2012.

8 McKenna, J.; Theodore, L.; personal notes; ETS, Inc.: Roanoke, VA, 1979.

9 Theodore, L.; personal notes; East Williston, NY, 2005.